AR-004-188

DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

MATERIALS RESEARCH LABORATORIES

MELBOURNE, VICTORIA

REPORT

MRL-R-940

AD-A151 416

A REVIEW OF ONE DIMENSIONAL SHAPED CHARGE THEORY
PART 1 - JET FORMATION

David A. Jones

Approved for Public Release

85 03 06 078

DEPARTMENT OF DEFENCE

MATERIALS RESEARCH LABORATORIES

REPORT

MRL-R-940

A REVIEW OF ONE DIMENSIONAL SHAPED CHARGE THEORY

PART 1 - JET FORMATION

David A. Jones

ABSTRACT

A comprehensive introduction to the basic physics of the collapse of a shaped charge liner and the formation of the associated jet and slug is presented. The hydrodynamic theory of the effect is treated in detail, and recent refinements of the theory are described. These include the calculation of a nonsteady Taylor angle formula, consideration of the inverse velocity gradient within the jet, and the effect of the compressibility of the liner material on jet formation.

Approved for Public Release

POSTAL ADDRESS: Director, Materials Research Laboratories
P.O. Box 50, Ascot Vale, Victoria 3032, Australia

SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED

DOCUMENT CONTROL DATA SHEET

REPORT NO.	AR NO.	REPORT SECURITY CLASSIFICATION
MRL-R-940	AR-004-188	Unclassified

TITLE

A Review Of The One Dimensional Shaped Theory
Part 1 - Jet Formation

AUTHOR(S)	CORPORATE AUTHOR
David A. Jones	Materials Research Laboratories P.O. Box 50, Ascot Vale, Victoria 3032

REPORT DATE	TASK NO.	SPONSOR
NOVEMBER, 1984	DST 82/192	DSTO

CLASSIFICATION/LIMITATION REVIEW DATE	CLASSIFICATION/RELEASE AUTHORITY
	Superintendent, MRL Physical Chemistry Division

SECONDARY DISTRIBUTION

Approved for Public Release

ANNOUNCEMENT

Announcement of this report is unlimited

KEYWORDS

Shaped charges
Jets

COSATI GROUPS 1901

ABSTRACT

This Australian report presents

~~A~~ comprehensive introduction to the basic physics of the collapse of a shaped charge liner and the formation of the associated jet and slug. ~~is presented.~~ The hydrodynamic theory of the effect is treated in detail, and recent refinements of the theory are described. These include the calculation of a nonsteady Taylor angle formula, consideration of the inverse velocity gradient within the jet, and the effect of the compressibility of the liner material on jet formation.

CONTENTS

A REVIEW OF ONE DIMENSIONAL SHAPED CHARGE THEORY

PART 1 - JET FORMATION

1. INTRODUCTION

The shaped charge warhead, based on the Munroe effect, has been used for the defeat of heavy armour since the second World War. A particularly comprehensive account of the historical development of these weapons has recently been given by Backofen [1].

Shaped charge warheads are currently in use by the Australian Army in the 66 mm M72L1A2 (LAW) and 84 mm Carl Gustaf anti-tank weapons, and it is anticipated that an advanced shaped charge device, the 103 mm MILAN, which incorporates wave shaping for increased performance, will soon be added to the list. Also, it seems highly likely that future torpedo designs will incorporate shaped charge warheads for defeat of submarine pressure hulls, as in the Stingray torpedo used by the Royal Navy.

MRL has been engaged on various investigations concerning shaped charge research, development and evaluation for some years. Examples of this work are the design and fabrication of a 38 mm copper lined shaped charge [2]. which has since become known as the MRL Standard Shaped Charge, and the recent use of this charge for a very illuminating investigation on the jet initiation and disruption of bare and covered Comp B and other explosives by M.C. Chick and D.J. Hatt [3].

Interest in the modelling of shaped charge performance has also been stimulated by the use at MRL during the last few years of several large two dimensional (2D) continuum mechanics finite difference codes [45-47]. Of these, one of the most useful for shaped charge work is the HELP code. This has recently been used by D.L. Smith to model the early stages of jet formation in the MRL 38 mm shaped charge, and a preliminary comparison with flash radiographs of the collapsing liner is extremely good [4]. Fig. 1 shows

a schematic illustration of the progressive stages of liner collapse and the formation of jet and slug, and Fig. 2 shows one of the recent MRL flash radiographs of the collapse of the MRL 38mm Standard Shaped Charge 18.5 μs after initiation of detonation.

Research on the basic understanding and refinement of shaped charge devices is currently being pursued in many weapons laboratories around the world, and much interesting work has been published in the open literature and in unclassified reports during the past decade. The purpose of this review is not only to acquaint the reader with these recent developments, but also to provide a reasonably detailed account of the basic hydrodynamic theory of shaped charges which was published in the late 1940's and early 1950's. It is hoped that this review will be of interest to people either using, designing or modelling shaped charges, as well as providing sufficient background material for the understanding and formulation of a one dimensional (1D) code for the analysis of shaped charge behaviour.

Modelling of classical shaped charges by the use of so called 1D codes has become quite common in the past few years. Possibly the best known example of such an approach is the BASC code used by the Ballistic Research Laboratory in the U.S.A., which has been extensively documented by Harrison [5]. Other codes include JETFORM, used by RARDE in the U.K. [6], DESC-1, designed by Carleone et al of Dyna East Corp [7], and the TB/ISL code of Hennequin in France [8]. As pointed out by Carleone et al [9], these codes are an invaluable addition to the overall strategy for the design of new shaped charge devices. Whilst the 2D codes are capable of providing very detailed information, they suffer from the disadvantage of requiring large computers and long run times, and as a consequence are not particularly well suited to extensive parametric studies. By contrast, the 1D codes evaluate simplified analytical expressions rather than a fully fledged finite difference scheme, which the name might suggest. 1D codes can be stored on the smallest of computers, require only seconds run time, and are ideal for parametric studies. An application of the ideas discussed in this report to the MRL Standard Shaped Charge is currently in progress, and will be reported shortly [10].

An outline of this report is as follows: section 2 describes the basic steady state hydrodynamic theory of jet formation published by Birkhoff, MacDougall, Pugh and Taylor in 1948 [11]. The description here is rather detailed as this paper provides the basic understanding of shaped charge behaviour. The theory was modified in 1952 by Pugh, Eichelberger and Rostoker to describe the stretching of the jet in free flight, and this resulted in the "non-steady state" theory of jet formation, commonly referred to as the PER theory [12]. This work is described in section 3. Before the PER theory can be utilised, expressions must be found for the liner velocity once it has been set in motion by the detonation, and a variety of methods for achieving this are described in section 4. The velocity of the jet tip is an important parameter in shaped charge work, and its calculation is complicated by an inverse velocity gradient within the leading portion of the jet, which causes the compression of the jet and the formation of a compact jet tip particle. The treatment of this effect is described in section 5. Finally, the early theories of Birkoff et al and Pugh et al assumed that the flow was incompressible, in which case a coherent jet must always be formed. The

modification to this result obtained by allowing the flow to be compressible is described in section 6.

This review is devoted entirely to the problem of jet formation, and no attempt has been made to include recent work on the important topics of jet breakup and jet penetration. It is intended that a review of such material will appear in a future MRL report.

2. STEADY STATE THEORY

As mentioned in the introduction, the first fairly complete mathematical theory for a classical shaped charge was published in the open literature in 1948 by Birkhoff, MacDougall, Pugh and Taylor [11]. Their analysis was based on the following assumptions:

(i) The pressure on the liner due to the explosive is so great that the yield strength of the metal is negligible, hence the metal will behave hydrodynamically. (Note that this does not imply that the metal is a liquid during the formation process).

(ii) The metal flow is non-viscous and incompressible.

(iii) The pressure on the collapsing liner does not increase the length of any portion of the liner.

(iv) The impulse from the detonation wave acts instantaneously, so that the acceleration of the liner can be neglected.

(v) Each element of the liner collapses with a constant velocity $\underline{V_o}$ which is independent of its original position along the cone axis.

Several of these assumptions have been modified or relaxed in more recent refinements of the theory, and these will be discussed in detail in later sections of this report. For the moment, we consider the original theory as presented by Birkhoff et al.

Consider Fig. 3 (adapted from reference [3]), which shows the detonation wave, with velocity $\underline{U_D}$, traversing the length of the cone. $\underline{PN}$ represents the velocity $\underline{V_o}$ of the liner. From assumption (v) we know that each element of the liner between A and P (the current position of the detonation front) has been compelled to move with the constant velocity $\underline{V_o}$. (ie $\underline{PN} = \underline{AA'}$), and from assumption (iii) we know that the length AP is equal to the length A'P. We now show that $\underline{PN}$ bisects the angle between the uncollapsed liner PB and the collapsing liner PM. As PN is parallel to AA' we have angle BPN = angle PAA', and angle NPA' = angle PA'A. Also, as AP = A'P, then angle PAA' = angle PA'A, ie angle BPN = angle NPA', which was to be proved.

Having found the direction of $\underline{V_o}$, we now derive an expression for its magnitude, again using assumptions (iii) and (v). Consider Fig. 4, which shows the detonation wave having advanced from P to P' in the time taken for the liner segment launched from P to hit the axis at N. Construct the line P'N. Again by assumption (iii), we have P'P ≡ P'N. Also construct the line PL, which is perpendicular to the original liner surface at P, and the line P'K, the perpendicular bisector of PN. Define angle NPL = δ_1 and PP'K = δ_2. It is easy to show, by summing angles at P' and P, that $\delta_1 = \delta_2 = \frac{1}{2}(\beta-\alpha)$,[2] where β is the angle which the collapsing liner makes with the axis, and α is the original cone half angle. From triangle P'PK we then have

$$\sin \delta = V_o/(2U), \tag{2.1}$$

where

$$U = U_D/\cos \alpha \ .$$

Equation (2.1) is fundamental to shaped charge work and was first derived by G.I. Taylor [14], and δ has since become known as the Taylor angle. It has recently been extended by Randers-Pehrson [15], and by Chou et al [16], to apply to non steady state conditions. These corrections will be discussed in detail in the next section.

Next we derive expressions for the velocities of the jet and slug. We denote the velocity of the collision point M by $\underline{V_1}$, and then move to a frame of reference in which M is at rest by imposing a uniform translational velocity of $-\underline{V_1}$ to $\underline{V_o}$. In this frame each segment of the liner PM then has a velocity $\underline{V_2}$ and moves along the line PM. Fig. 5 illustrates the velocity relationship. Applying the sine rule we have

$$\frac{V_o}{\sin \beta} = \frac{V_2}{\sin [\frac{\pi}{2} - (\alpha + \delta)]} = \frac{V_1}{\sin [\frac{\pi}{2} + (\alpha + \delta - \beta)]}$$

from which we find

$$V_2 = V_o \cos (\alpha + \delta)/\sin \beta \tag{2.2}$$

$$V_1 = V_o \cos (\beta - \alpha - \delta)/\sin \beta \tag{2.3}$$

Because of assumption (v), an observer in the frame of reference moving with the velocity $\underline{V_1}$ will see the steady state situation depicted in Fig. 6, ie. two fluid jets with velocities $\underline{V_2}$ colliding at an angle 2β. This situation has been described by Milne-Thomson [17], and the result of the collision will be two fluid streams, a "jet" moving to the right with velocity $\underline{V_2}$, and a "slug" moving to the left, also with velocity $\underline{V_2}$. Hence, in the stationary system of coordinates, the jet will be moving to the right with a velocity given by

$$V_j = V_1 + V_2 \tag{2.4}$$

while the slug will also be moving to the right with the slower velocity

$$V_s = V_1 - V_2 \qquad (2.5)$$

Using equations (2.2) and (2.3) in (2.4) and (2.5) we find

$$V_j = V_o \cos(\alpha + \delta - \beta/2)/\sin(\beta/2), \qquad (2.6)$$

$$V_s = V_o \sin(\alpha + \delta - \beta/2)/\cos(\beta/2). \qquad (2.7)$$

For the simple case treated here in which $\underline{V_o}$ is independent of position along the liner we have already found

$$\delta = \tfrac{1}{2}(\beta - \alpha), \qquad (2.8)$$

in which case equations (2.6) and (2.7) take the simple form

$$V_j = V_o \cos(\alpha/2)/\sin(\beta/2), \qquad (2.9)$$

$$V_s = V_o \sin(\alpha/2)/\cos(\beta/2). \qquad (2.10)$$

The division of liner mass between jet and slug is governed by the conservation of momentum. Let m be the liner mass per unit length approaching the junction. Let m_j be that part of m going into the jet and m_s be that going into the slug. By equating the horizontal components of momentum before and after passing the stagnation point M in the moving frame we have

$$mV_2 \cos\beta = m_s V_2 - m_j V_2, \qquad (2.11)$$

while from conservation of mass

$$m = m_s + m_j \qquad (2.12)$$

Combining (2.11) and (2.12) we find

$$m_j/m = \sin^2(\beta/2), \qquad (2.13)$$

$$m_s/m = \cos^2(\beta/2). \qquad (2.14)$$

It is important to point out that this steady state analysis is strictly true only for the case of a wedge shaped liner. Consider the cross-section of the charge shown in either Fig. 3 or Fig. 4. The wedge shaped, or linear charge, is defined by supposing the cross-section to be the same for an infinite distance perpendicular to the cross-section, while the conical case is obtained by rotating the cross-section around the line of symmetry. In order for the whole of the collapse process to appear stationary and the simple situation depicted in Fig. 6 to be applicable it is necessary for the total mass per unit distance along the axis to be constant. This is true for a wedge shaped charge, but is only approximately true for a conical liner of constant thickness. Nevertheless, the application of this theory to conical liners is remarkably accurate, as we will see in later sections.

In summary, the steady state model predicts that both the velocities and cross-sectional thickness of the jet and slug will be constant. Experimentally however this is known not to be the case: the jet in particular has a marked velocity gradient along its length, the tip of the jet travelling some four to five times faster than the base, which leads to considerable elongation of the jet in flight before target penetration. This fact was known by Birkhoff et al and was used by them in their description of jet penetration, but was not included in their theory of jet formation. The modification to the simple steady state theory to account for the gradient in jet velocity was made by Pugh, Eichelberger and Rostoker [12] and is described in the next section.

3. NON STEADY STATE THEORY

The velocity gradient within the jet was satisfactorily explained by Pugh et al [12] by modifying assumption (v) as follows:

The velocities with which the various elements of the cone liner collapse when they are struck by the detonation wave depends upon the original position of the element in the cone.

This is certainly a reasonable assumption to make as the ratio of the explosive mass to liner mass decreases continuously from apex to base and, bearing the Gurney formulae in mind [18], we would expect a corresponding decrease in liner velocity (remember that we are still assuming that the period of acceleration is negligible). The decrease in velocity of each segment hitting the axis results in a corresponding decrease in velocity of each jet element as it is formed, hence the velocity gradient within the jet is explained.

Because V_o is a function of position along the cone axis it is now impossible to find a set of coordinates in which the collapse appears as a true steady state process. This means, in particular, that it is no longer possible to use Bernoulli's theorem to simplify the hydrodynamics, as discussed by Birkhoff et al, and which is fundamental to the simple description of the collapse as pictured in Fig. 6. Pugh et al overcome this problem by using the same approach as Birkhoff et al, but applying it to a succession of individual zonal elements. Each element is then considered independently in the appropriate constant velocity coordinate system. As a result, equations (2.6) and (2.7) for the velocities of jet and slug remain valid in the non steady state theory, where now V_o is a monotonic decreasing function of x, where x measures distance along the cone axis. Equations (2.13) and (2.14) for the distribution of mass between jet and slug also remain valid in the new theory.

Another consequence of the decrease in liner velocity with distance along the axis is that the collapse angle β is no longer constant, but increases as the collapse proceeds. This is easily explained by referring again to Fig. 4. Consider the element at Q, if this collapsed with the same

velocity as the element from P, then it would reach S when the element from P reached N, and the collapsing segment of liner NSP' would form a straight line. However, as the collapse velocity at Q is less than that at P the collapsing liner adopts the shape NRP', which means that the angle β which the liner makes with the axis at N is greater than the angle β^+ which it would have made with the axis if the collapse velocity were constant.

To calculate the variation of β with distance along the cone Pugh et al adopt the following procedure. Let the cylindrical coordinates of R in Fig. 4 be (r, z) and the coordinates of the original position Q of R in the liner be (x tan α, x). The connection between the two sets of coordinates is given by

$$z = x + V_o (t - T) \sin A \tag{3.1}$$

$$r = x \tan \alpha - V_o (t - T) \cos A \tag{3.2}$$

where t is the time (t = o corresponds to the detonation wave passing the apex of the cone), $T = x/U_D$, and $A = \alpha + \delta (x)$. The slope of the collapsing liner is given by $\partial r/\partial z$, and the collapse angle β is defined by

$$\tan \beta = \left(\frac{\partial r}{\partial z}\right)_{r=o} . \tag{3.3}$$

From equations (3.1) and (3.2) we have

$$\frac{\partial r}{\partial z} = \frac{\tan \alpha - V_o'(t - T) \cos A + V_o \cos A/U_D + V_o A' (t - T) \sin A}{1 + V_o' (t - T) \sin A - V_o \sin A/U_D + V_o A' (t - T) \cos A} \tag{3.4}$$

where the prime denotes differentiation with respect to x. The condition r = o is equivalent to

$$t - T = x \tan \alpha/V_o \cos A . \tag{3.5}$$

Substituting (3.5) and (3.4) into (3.3) we find

$$\tan \beta = \frac{\sin \beta^+ - x \sin \alpha (1 - \tan A \tan \delta) V_o'/V_o}{\cos \beta^+ + x \sin \alpha (\tan A + \tan \delta) V_o'/V_o} \tag{3.6}$$

where $\beta^+ = \alpha + 2\delta$ and is equal to the value of the collapse angle in the steady state case. From the discussion already given we know that V_o will be negative, while the expression in the brackets in equation (3.6) will be positive for usual shaped charge dimensions, hence equation (3.6) shows that β will increase as x increases.

It should be noted here that the analysis presented above applies only to the case of a plane detonation wave. Generalizations of these results have been made by Allison and Vitali [19] for point detonation, by the present author for a spherically converging detonation wave [20], and by Behrmann for an arbitrarily shaped detonation wave and liner [21].

An interesting development of equation (3.6) has also recently been made by Hirsch [22]. Using simple trigonometric identities he has shown that the equation for tan β can be written in the form

$$\tan \beta = \tan (\beta^{+} + \Delta\beta), \tag{3.7}$$

where Δβ is defined by

$$\tan \Delta\beta = \big(- x \sin \alpha/(\cos A \cos \delta)\big)(V_o'/V_o). \tag{3.8}$$

This simple development clearly shows how the velocity gradient of the liner continuously increases the collapse angle β to values greater than the steady state value β^{+}. The introduction of the angle Δβ also simplifies the expression for the velocity of a liner segment in the coordinate system moving with the collapse point (equation 2.3), and when this is considered with the criterion for coherent jet formation (which will be discussed in section 6) it allows Hirsch to make some interesting observations on the effect of confinement on the jet tip origin and its variation with α.

The derivation of the Taylor angle formula, eq (2.1), clearly shows that this relation is valid only under steady-state conditions. An extension of this formula to non steady conditions has been made empirically by Randers-Pehrson [15], and analytically by Chou et al [16]. Both approaches take into account not only the gradient in liner velocity along the length of the liner, but also the finite acceleration of each liner segment.

Randers-Pehrson used a simplified 2D computer code to model the acceleration of the metal liner by the explosive. The metal was assumed to behave as a fluid and its thickness was ignored. Each point along the metal surface was assumed to accelerate according to the equation

$$V = V_o \,(1 - \exp - (t - T)/\tau), \tag{3.9}$$

where V is the velocity at any instant, V_o is the final velocity, T is the time at which the detonation front reaches the element, and τ is the time constant for the acceleration. V_o, T and τ all vary with initial location. After many simulations it was found that the projection angle δ could be described by the expression

$$\sin \delta = \frac{V_o}{2U} - \frac{V_o'\tau}{2} - \frac{(V_o'\tau)^2}{5}, \tag{3.10}$$

where V_o' is now defined as being the derivative of V_o with respect to distance along the surface. Typical values for V_o' and τ are

.01 (mm/μs)/mm and 1 to 5 microseconds respectively, so the effect of these correction terms on δ can be quite significant.

Chou et al approached the problem analytically by writing the equations of motion of each liner segment in the form

$$\dot{\delta} = \frac{\dot{V}}{V} \tan(\theta - \delta), \tag{3.11}$$

$$\dot{\theta} = -V' \cos(\theta - \delta), \tag{3.12}$$

where θ is the angle between the original liner contour and the current liner contour at l (l is the length coordinate along the liner contour) and the dot denotes differentiation with respect to time. Assuming θ-δ is small, equations (3.11) and (3.12) yield

$$\delta = -\int_T^t V'dt + \frac{1}{2V}\int_T^t (V^2)'dt. \tag{3.13}$$

When equation (3.9) is used to evaluate this expression the final result becomes

$$\delta = \frac{V_o}{2U} - \frac{1}{2}\tau V_o' + \frac{1}{4}\tau' V_o \tag{3.14}$$

Note that the first correction term in equation (3.14) is the same as found by Randers-Pehrson. Chou et al have compared equation (3.14), (3.10) and (2.1) with the projection angle calculated from the 2D Lagrangian code TEMPS. The model system used was the BRL 81.3 mm diameter, 42°, conical shaped charge with a copper liner 1.9 mm thick. Fig. 7 shows the results. The differences between the two non-steady expressions for δ are minimal, and clearly agree much better with the TEMPS calculation than the original Taylor expression.

4. CALCULATION OF LINER IMPULSE

Before the non-steady jet formation theory described in the previous section can be used to model a real shaped charge we need some means of calculating the velocity imparted to the liner by the explosive loading, i.e. we need to know the speed V_o and projection angle δ as a function of length along the liner. As we have just seen, the non-steady Taylor angle formula provides a reasonably accurate relationship between these two variables, so that in principle we need determine only V_o or δ. Both approaches are used in shaped charge design.

The most commonly used method of calculating the projection angle for a given shaped charge geometry is by use of the Richter approximation [23] as modified by Defourneaux [24]. This has the form

$$\frac{1}{2\delta} = \frac{1}{\phi_o} + K\rho\left(\frac{\varepsilon}{e}\right), \qquad (4.1)$$

where K and ϕ_o are empirical constants which depend on the explosive and the angle at which the detonation wave intersects the liner. They can be determined from experiments with explosive-metal slabs or sandwiches, and can also be found by fitting to a small subset of the shaped charge collapse data. ρ and ε are the density and thickness of the liner material and e is the explosive thickness. Harrison [5] has modified equation (4.1) to take account of the confinement of the casing around the charge. He writes (4.1) in the form

$$\frac{1}{2\delta} = \frac{1}{\phi_o} + \frac{K\rho\varepsilon}{Be}, \qquad (4.2)$$

where B is a correction factor given by

$$B = 1 + A/(\rho_c e_c). \qquad (4.3)$$

ρ_c and e_c are the density and thickness of the casing, and A is a constant which is determined from experiment. Carleone and Chou [25] have also modified equation (4.1) to take into account the non planar geometry of conical shaped charges. They replaced the thickness ratio ε/e by the area ratio A_ε/A_e, where A_ε and A_e are the surface areas generated by rotating the lines representing the thickness ε and e about the cone axis for each point x. An example of the accuracy of this type of approach, when coupled with the PER theory of jet formation, is given in Fig. 8. This shows a comparison of experimental and theoretical jet velocity distributions for the heavily-confined 42° conical shaped charges reported by Di Persio et al [26]. It is interesting to note that the respectable agreement between theory and experiment is obtained using only the steady state Taylor equation for δ, and by neglecting the variation of K and ϕ_o with the angle of incidence of the detonation wave on the liner. The values used were $K = 0.25\ \mathrm{cm^3/g}$ and $\phi_o = 23^0$.

An alternative approach to the problem of calculating the liner impulse is to first use a form of Gurney equation to calculate V_o and then use one of the Taylor equations to calculate δ. The original Gurney model [18] (see also [27] for a discussion of this work) calculated the asymptotic metal velocities as a function of the explosive to metal mass ratio C/M for symmetrical geometries using an energy equation and the assumption of a linear velocity gradient in the explosive gases, and could also treat the plane asymmetric case by including a momentum balance equation. The formulae have the general form

$$V_o/\sqrt{2E} = f(C/M). \qquad (4.4)$$

where $\sqrt{2E}$ is an empirical constant known as the Gurney energy, and the form of the function f depends on the geometry of the explosive-metal system. This standard Gurney approach is unsuitable for modern shaped charge design in several respects. We saw in the previous section that the non-steady Taylor angle calculation requires that the full velocity-time curve for each liner segment be known, while the standard Gurney model calculates only the asymptotic velocity V_o. Also, the standard model applies only to explosive systems, and needs to be modified to handle the implosive case. Several papers have appeared in the last few years which extend the Gurney model and attempt to remove these deficiencies.

Chou et al [16] have considered the problem of a steady-state imploding cylinder using the same general assumptions as the classical Gurney theory. Because of the curved geometry in the imploding case the momentum equation contains an extra term which represents an outward impulse applied to the explosive gases. This extra term is approximated in terms of the Chapman-Jouget pressure, the characteristic acceleration time, and the inner and outer radii of the cylinder. The expression for the final liner velocity as a function of charge-to-mass ratio is then compared with the original Gurney formula and a 2D code calculation, and is found to be a significant improvement on the standard result.

The recent paper by Chanteret [28] extends the Gurney method in the two areas needed for application to accurate shaped charge calculations. An analytical model is developed for energy partitioning during expansion which allows a calculation of the Gurney energy as a function of the expansion ratio; when combined with the Taylor angle equation this allows the entire velocity-time curve for all steady state geometries to be calculated. The results are found to agree well with 2D code calculations or with available experimental results. Chanteret also considers the problem of implosive geometries by introducing the concept of a fictitious rigid boundary in the explosive and applying this to the case of the imploding cylinder considered by Chou et al. A comparison of liner velocities as a function of charge to mass ratio C/M shows that the two approaches agree remarkably well.

Hennequin [29] has also developed an analytical model of liner collapse based on assumptions similar to those of Gurney. The laws of conservation of mass, momentum and energy are used and applied in detail to the explosive gases, liner and casing. Both cylindrical and conical geometries are considered.

For the conical case, a 60° apex angle was used and liner velocity as a function of time for different elements along the length of the cone was compared with 2D hydrocode calculations using the HEMP code. The results agreed well for the first 1-2 μs, but then showed disagreements of the order of up to 40%. Nevertheless, the results are encouraging, and undoubtedly work will continue in this direction.

A more detailed approach to the calculation of liner impulse has been made by Kiwan & Wisniewski [30]. Their idea is to replace the liner by a system of discrete solid liner elements and then to assume that both the strength properties of the liner material and the interaction forces between

liner elements can be neglected. The impulse transmitted by the detonation products to a liner element is then obtained by integrating the excess pressure along the path of the liner element. The pressure wave behind the detonation front is approximated by the following expression

$$P(z) = P_f - P_o + (P_f - P_o)(z - 1)/(1 - \lambda_o),$$

$$\lambda_o < z < 1 \qquad (4.5)$$

where $z = x/U_D t$, P_f is the detonation pressure, P_o the ambient atmospheric pressure and λ_o is the value of z for which the particle velocity vanishes in the Taylor wave. Equation (4.5) applies only to a one dimensional plane Taylor wave in a tube with an open end which implies a completely confined charge. Kiwan & Wisniewski show how this can be modified to apply to an unconfined charge, and also to take account of the cavity in the explosive which causes a reduction in the total available energy. The liner motion is then obtained by numerically integrating Newton's equations for each liner element, i.e.

$$\sigma \ddot{x}^k = P(x^k(t), Y^k(t), t) \sin \alpha \qquad (4.6)$$

$$\sigma \ddot{y}^k = P(x^k(t), Y^k(t), t) \cos \alpha \qquad (4.7)$$

where k labels the liner element, P is given by an equation similar to (4.5), σ is the mass per unit area of the liner, and the dot indicates differentiation with respect to t. The jet formation equations used are similar to those of the PER theory with $\delta = 0$, as the above approach implicitly assumes that each element of the liner collapses in a direction perpendicular to the surface of its initial position on the liner. The theory has been applied to two shaped charges, having diameters of 3.44" and 3.60" and apex angles of 42° and 60° respectively. Both charges are loaded with Comp. B and have .106" copper liners. The calculated results are in good agreement with the experimental results of Di Persio, Whiteford and Simon [31].

Kiwan & Wisniewski [30] have remarked that a possible improvement for their model would be to compute the pressure wave using one of the many available hydrocodes and then to couple this calculation to their equations of motion for the liner segments. A similar approach has been used by several groups in the past few years with varying degrees of success. Edwards & Godfrey [32] describe a method based on the CHAMP computer code system in which the high-explosive detonation and hydrodynamics are computed by an explicit Eulerian finite difference method while the metal liner is viewed as a polygonal line of mass segments in which the metal volume and internal stresses are neglected. The formation of the jet is treated analytically using the PER theory. This approach has also been used by S.L. and H. Hancock [33]. Van Thiel and Levitan [34] describe a method in which a Lagrange code (HEMP) is used to calculate the motion of the detonation products and liner segments, which are then coupled to analytic expressions

similar to those of PER to describe the jet formation. Kivity et al [35] also describe a scheme in which the explosive and liner are treated by the DISCO code (a Lagrange code similar to HEMP), while the jet formation is described by the PER theory. A similar approach is also used at the Ballistic Research Laboratories [36].

5. INVERSE VELOCITY GRADIENT EFFECT AND JET TIP FORMATION

The PER theory is based on the assumption of a negligible acceleration time, i.e. it is assumed that the detonation impulse instantaneously accelerates the liner segment to its terminal velocity V_o. In practice this does not occur. We saw in the previous section that each liner segment follows a velocity-time curve which can be reasonably approximated by an equation of the form of (3.9), where the time constant τ can have a value of several microseconds. This means that liner segments near the apex of the cone will not have sufficient time to be accelerated to their terminal velocity V_o before colliding on the cone axis. As the distance between the liner segment and the apex of the cone increases, more space (time) is available for the segments to be accelerated. Eventually a point is reached at which the segments do have sufficient time to be accelerated to V_o, but then the influence of the decrease of the explosive mass to liner mass takes over and the value of V_o decreases as x increases. The point at which this usually occurs is situated about one third of the way along the cone axis from the apex, but the exact position depends on the geometry used. More information on this can be found in reference [49]. The net result is that the velocities of the liner segments just before collision with the axis first increase to a maximum value somewhere near the midpoint of the cone, then decrease. The jet segments produced after collision of the liner segments on the axis will also follow this pattern. A further limitation on the performance of the shaped charge occurs in the base region where the combined effect of a significant drop in C/M ratio and the explosive thickness droping below the critical thickness to support detonation produce jet elements with velocities too low to contribute to penetration of target material. This is described more fully in reference [49].

The velocity distribution within the jet as just outlined will not be stable of course. In the leading part of the jet a particular jet element will not travel for long before the jet element behind it, travelling at a higher velocity, will collide with it and the two will combine to form a single particle moving with a velocity intermediate between the velocities of the two particles before collision. This process of collision and jet compression will lead to the formation of a relatively massive single tip particle, and will terminate when the velocity of the tip particle is greater than the velocity of the nearest following jet segment. This phenomenon has been known experimentally for some time [31], and the presence of a massive jet tip is clearly evident in flash radiographs. The paper by Perex et al provides a good illustration of this [37], as does the MRL flash radiograph reproduced in Fig. 2. Note that the high quality of this radiograph also allows the ablation of the jet tip particle to be observed. A similar compression occurs for the slug, as has been noted by Kiwan and Wisniewski [30].

Several authors have successfully modelled the formation of a compact jet tip within the 1D approximation. Carleone et al [38] have made both an experimental and theoretical study of this effect. They describe a series of experiments using the 81.3 mm BRL Standard Charge in which the apex end of the liner was filled to different heights with Woods metal, an inert material which inhibits the collapse of the apex portion of the liner. Several radiographs of each jet were taken and used to observe the jet tip particle and to measure the jet velocity. No effort was made to directly examine the collapse process and the fate of the woods metal however, which could effect the interpretation of the data. The experimental results were compared with a 1D theory based on the PER equations, but containing some important modifications, the most important of these being the inclusion of liner acceleration. The liner elements were still assumed to travel along a straight line from the liner to the axis, but each element was assumed to have a finite constant acceleration until it either hit the cone axis or attained the final collapse velocity predicted by the (steady state) Taylor relation. It was assumed that the jet elements experienced a perfectly plastic collision during the compression stage, and the velocity of the tip particle was then calculated using conservation of momentum. Experimental and theoretical results were then compared and showed quite good agreement for jet tip velocity versus percentage of ineffective cone height, jet velocity versus jet element position, and jet radius versus jet segment position.

Harrison [5] has described an iterative scheme for the calculation of the inverse gradient effect. The bending angle ϕ (which is twice the Taylor angle δ) is calculated from the modified Defourneaux equation (equation (4.2)). A new equation for the bending angle is then defined as follows:

$$\phi_N = \phi e \,(b/\tau^*)^{1/2} \tag{5.1}$$

where τ^* is the time which an element takes to reach the axis, e is the thickness of the explosive, and b is related to the density and thickness of both the liner and explosive. The iteration is carried out between equation (5.1) and the expression for the collapse time τ^* until ϕ_N approaches ϕ to within some desired degree of accuracy. The scheme then proceeds in a similar manner to the calculation described by Carleone et al [38]. Harrison makes an extensive comparison between this simple 1D theory, a 2D hydrocode (the BRL version of the HELP code) and experiment for a variety of different shaped charges and generally finds quite good agreement.

Kiwan and Wisniewski [30] have also made a detailed calculation of the "pile up" of the leading jet elements to form a jet tip particle, but their comparison with other calculations, or with experiment, is nowhere near as extensive as either Carleone et al or Harrison. One novel feature of their calculation is that they allow the collisions in the jet compression region to be either purely elastic or purely plastic. This makes a difference of approximately 10% to the final jet tip velocity.

6. EFFECT OF COMPRESSIBILITY ON JET FORMATION

The original steady state shaped charge theory of Birkhoff et al, and its extension to the non steady case by Pugh et al, both assumed that the liner material flowed as an incompressible fluid. The result of this assumption of incompressibility is that a coherent jet must always be formed. If the fluid is compressible however the situation is more complicated, and there is the possibility of either a no jet configuration, or the formation of a non cohesive jet [39].

The 2D wedge shaped geometry formed by the angular impact of two plates at high velocity has been analysed in detail by Walsh, Shreffler and Willig [39], and by Cowan and Holtzman [40]. They find that if the fluid velocity of the collision is subsonic (relative to a frame moving with the velocity of the collision point) then a coherent jet will always be formed, as in the incompressible case. For supersonic collisions however jetting only occurs if $\beta > \beta_c$, where β_c is the critical angle at which the shock wave at the stagnation point becomes detached (the angle between the plates is 2β). For $\beta < \beta_c$ no jetting occurs.

These conclusions have been verified by Harlow and Pracht [41] by numerical calculations of the oblique collapse of two metal plates using the Particle-in-Cell method. These calculations were also able to include the effect of viscosity on the collapse process and were also able to examine the transient stages leading to the formation of the steady state.

Chou, Carleone and Karpp [42,43] have reviewed this earlier work and considered its application to the axially symmetric geometries which occur in conically lined shaped charges. They note that the criterion given by Walsh et al and by Cowan and Holtzman is based on the maximum angle for an attached shock at the collision point as determined by the shock polar for the material. As the shock conditions and shock polar are applicable to curved shocks at individual points on the shock, they therefore believe that the above jetting criterion should also be applicable to a shaped charge with a conical liner. They therefore propose the following:

Jetting Hypothesis

1. For subsonic collisions, a solid coherent jet always occurs.

2. For supersonic collisions, jetting occurs if $\beta > \beta_c$ for a given v; the jet formed is not coherent.

3. For supersonic collisions, if $\beta < \beta_c$ for a given v, no jet will be formed.

(Here v is the velocity of the liner in the frame moving with the velocity of the collapse point). The remainder of the report by Chou et al [42] details a large number of experiments and numerical computations which are in agreement with their hypothesis.

A similar jetting hypothesis has been used by Harrison in the BASC code [5]. His criterion for a coherent jet is that the velocity v should satisfy the inequality

$$v < 1.23\ C \qquad (6.1)$$

where C is the sound velocity in the liner material and the numerical factor has been determined by comparing jet radiographs with BASC results.

Finally, we note that an interesting application of this work has recently been made by Carleone and Chou [44] in calculating a theoretical maximum velocity for a coherent shaped charge jet. Using the above jetting hypothesis, the simple Gurney model, and some geometrical arguments, they derive the following upper limit for jet velocity

$$V_j < 2.41\ C \qquad (6.2)$$

Carleone and Chou compare this criterion with a large amount of published and unpublished shaped charge data and conclude that the maximum jet tip speed given by equation (6.2) has not been exceeded by any cohesive jet.

It is interesting to note that the application of equation (6.2) to a steel liner results in a theoretical V_{jmax} of 9.8 mm/μs (the value for copper is 10.2mm/μs), whereas Mader et al have recently reported the formation of a jet from a 4.0 mm thick hemishell having a jet trip velocity of 18mmμs [48]. It is uncertain however whether the theory of Carleone and Chou is applicable to this geometry.

7. ACKNOWLEDGEMENTS

I would like to thank F.G.J. May for many illuminating discussions on the behaviour of shaped charges, and D.L. Smith and M.C. Chick for providing many useful references and suggestions.

8. REFERENCES

1. Backofen, J.E. Jr., "The Weaponization of Shaped Charge Technology", 4th Int. Symp. on Ballistics, Oct. 1978.

2. Chick, M.C. and Briggs, G.P. "Fabrication and Performance Assessment of Shaped Charges", MRL Report No. 546, June 1974.

3. Chick, M.C. and Hatt, D.J. "Metal Jet Initiation of Bare and Covered Explosives; Summary of the Mechanism, Empirical Model and Some Applications". MRL-R-830, October 1981.

4. Smith, D.L. unpublished work, 1983.

5. Harrison, J.T. "BASC, An Analytical Code for Calculating Shaped Charge Properties". 6th Int. Symp. on Ballistics, October 1981.

6. Cullis, I.G. RARDE, Private Communication, 1983.

7. Carleone, J., Chou, P.C. and Tanzio, C.A. Dyna East Corp Tech Rep DE-TR-75-4 (1975).

8. Hennequin, E., Pfeffer, G. and Winaver, A. "Numerical Simulation of Shaped Charge Detonation". 3rd Int. Symp. on Ballistics, 1977.

9. Carleone, J., Chou, P.C. and Simpson, R. "A Synthesis Model for Modern Warhead Design", 4th Int. Symp. on Ballistics, October 1978.

10. Jones, D.A. "Modelling the MRL 38 mm Shaped Charge using a simple ID Code". MRL Report in preparation.

11. Birkhoff, G., MacDougall, D.P., Pugh, E.M. and Taylor, G. "Explosives with Lined Cavities". J. Appl. Physics 19 563-582 (1948).

12. Pugh, E.M., Eichelberger, R.J. and Rostoker, N. "Theory of Jet Formation by Charges with Lined Conical Cavities". J. Appl. Physics 23, 532-542 (1952).

13. Johnson, W. "Impact Strength of Materials". (Edward Arnold Ltd, London, 1972).

14. Taylor, G.I. "Analysis of the Explosion of a Long Cylindrical Bomb Detonated at One End". (1941), Scientific Papers of G.I. Taylor, Vol 3, Cambridge Univ. Press, 1963, pp. 277-286.

15. Randers-Pehrson, G. "An Improved Equation for Calculating Fragment Projection Angle". Proceedings of Second International Symposium on Ballistics, Florida, March 1977.

16. Chou, P.C., Carleone, J., Hirsch, E., Flis, W.J. and Ciccarelli, R.D. "Improved Formulas for Velocity, Acceleration and Projection Angle of Explosively Driven Liners". Proceedings of 6th Int. Symp. on Ballistics, pp. 262-272, October 1981.

17. Milne-Thomson, L.M. "Theoretical Hydrodynamics", MacMillan and Co., 1938.

18. Gurney, R.W. "The Initial Velocity of Fragments from Bombs, Shells and Grenades", BRL Report No 405, Sept 1943.

19. Allison, F. and Vitali, R. "An Application of Jet Formation Theory to the 105-mm Shaped Charge", BRLR-1165, March 1962.

20. Jones, D.A. unpublished work, 1983.

21. Behrmann, L. "Calculation of Shaped Charge Jets using Engineering Approximations". Physics International Co. AFATL-TR-73-160 Vol 1, August 1973.

22. Hirsch, E. "A Simple Representation of the Pugh, Eichelberger and Rostoker solution to the shaped charge jet formation problem", J. Appl. Phys. 50 4667 (1979).

23. Richter, H. "On the theory of shaped charges : Motion of thin liners of plastic material on the surface of a plane explosive", Note Technique ISL No. 6a/48 (1948).

24. Defourneaux, M. "Theorie Hydrodynamique des Charges Creuses", Memorial de l'Artillerie Francaise, T.44, 2^{e} Fasc., pp. 293-334, 1970.

25. Carleone, J. and Chou, P.C. "A One Dimensional Theory to Predict the Strain and Radius of Shaped Charge Jets". Proceedings of the First International Symposium on Ballistics, Orlando, Florida, November 1976.

26. Di Persio, R., Simon, T. and Martin, T.H. "A Study of Jets from Scaled Conical Shaped Charge Liners", BRL Memorandum Report No 1298, August 1960.

27. Jones, G.E., Kennedy, J.E. and Bertholf, L.D. "Ballistics calculations of R.W. Gurney, Am. J. Phys. 48 264 (1980).

28. Chanteret, P.Y. "An Analytical Model for Metal Acceleration by Grazing Detonation", Proceedings of the 7th International Symposium on Ballistics, April 1983, 515.

29. Hennequin, E.M. "Analytical Model of the Shaped Charge Liner Collapse", Proceedings of the 7th International Symposium on Ballistics, 525, April 1983.

30. Kiwan, A.R. and Wisniewski, H. "Theory and Computations of Collapse and Jet Velocities of Metallic Shaped Charge Liners", BRL Report No 1620, November 1972.

31. Di Persio, R., Whiteford, C.W. and Simon, J. "An experimental method of obtaining Collapse Velocities of the Inner Walls of a Linear Shaped Charge Liner". BRL Memorandum Report No 1696 (1965).

32. Edwards, L.L. and Godfrey, C.S. "Computer Code Design of Shaped Charges", Lawrence Livermore Laboratory Report No UCRL - 52598, October 1978.

33. Hancock, S.L. and Hancock, H. "A Coupled Eulerian-Mass Point Method for Calculating Shaped Charges", TCAM 76-7, Physics International Co., San Leandro, CA (1976).

34. van Thiel, M. and Levitan, J.A. "Jet Formation Experiments and Computations with a Lagrange Code", J. Appl. Phys. 51 6107 (1980).

35. Kivity, J., Mayseless, M., Hirsch, E. and Feder, M. "An Efficient Model for Shaped Charge Design", Proceedings of 7th International Symposium on Ballistics, p.231, April 1983.

36. Grace, F. and Menne, D.F. Private Communication, January 1984.

37. Perez, E., Fauquignon, C. and Chanteret, P. "Fundamental Studies of Shaped Charge Mechanisms", 3rd Int. Symp. on Ballistics, 1977.

38. Carleone, J., Jameson, R. and Chou, P.C. "The Tip Origin of a Shaped Charge Jet", Propellants and Explosives 2, 126-130 (1977).

39. Walsh, J.M., Shreffler, R.G. and Willig, R.J. "Limiting Conditions for Jet Formation in High Velocity Collisions". J. Appl. Phys. 24, 349 (1953).

40. Cowan, G.R. and Holtzman, A.H. "Flow Configurations in Colliding Plates: Explosive Bonding". J. Appl. Phys. 34 928 (1963).

41. Harlow, F.H. and Pracht, W.E. "Formation and Penetration of High-Speed Collapse Jets". Physics of Fluids 9, 1951 (1966).

42. Chou, P.C., Carleone, J. and Karpp, R.R. "Study of Shaped Charge Jet Formation and Breakup", U.S. Army Ballistic Research Laboratory Contract Report No 138, 1974.

43. Chou, P.C., Carleone J. and Karpp, R.R. "Criteria for Jet Formation from Impinging Shells and Plates". J. Appl. Phys. 47 2975 (1976).

44. Carleone, J. and Chou, P.C. "Maximum Jet Velocity and Comparison of Jet Breakup Models". U.S. Army BRL Contract Report No ARBRL-CR-00458, July 1981.

45. Smith, D.L. "DEGAS - A Subroutine to isolate material packages in the HELP hydrodynamic code". MRL-R-872, February 1983.

46. Smith, D.L. "Speeding up the HELP hydrodynamic code". MRL-R-925, May 1984.

47. Waschl, J.A. "Shock wave modelling and the HULL computer code - Formation of Mach Stems". MRL-R-907 November 1983.

48. Mader, C.L., Pimbley, G.H. and Bowman, A.L. "Jet Penetration of Inerts and Explosives", Los Alamos Report No. LA-9527-MS, 1982.

49. Grace, F.I. "Impact of High Explosives Characteristics on Shaped Charge Design and Performance", TTCP Sub group W, Action Group 9. Lightweight Anti-Tank Weapons, Final Report, Vol. II, p. 131, November 1983.

GLOSSARY OF SYMBOLS

$\underline{U}_D$	Velocity of detonation wave
$\underline{V}_o$	Final collapse velocity of liner
α	Original cone half angle
β	Generic symbol for collapse angle
β^+	Collapse angle in Steady State theory
U	$U_D/\cos\alpha$
δ	Taylor angle
$\underline{V}_1$	Velocity of stagnation point
$\underline{V}_2$	Velocity of liner in frame moving with velocity of stagnation point
V_j	Speed of jet
V_s	Speed of slug
m	Mass per unit length of liner
m_j	Mass per unit length of jet at stagnation point
m_s	Mass per unit length of slug at stagnation point
x	Measures distance along cone axis from apex
r,z	Cylindrical coordinates of typical liner element
T	Time for detonation wave to travel distance x
A	$\alpha + \delta$
V'_o	Derivative of V_o with respect to x or ℓ, where appropriate
$\Delta\beta$	Difference between β and β^+
V	Liner velocity when finite acceleration time is considered
τ	Time constant for acceleration of liner
θ	Angle between original liner contour and current liner contour
ℓ	Length corrodinate along liner contour
$\dot{\theta}$	Derivative of θ with respect to time

K, ϕ_o	Empirical constants used in Richter approximation for δ
ρ	Density of liner material
ε	Thickness of liner material
e	Thickness of explosive
B	Correction factory for Richter approximation to take casing into account
ρ_c	Density of casing material
e_c	Thickness of casing material
A	Experimentalloy determined constant for use in modified Richter approximation
A_ε, A_e	Surface areas generated by rotating the lines representing thicknesses ε and e about the cone axis
C/M	Explosive to metal mass ratio
$\sqrt{2E}$	Gurney energy
P	Pressure wave behind detonation front
P_f	Deontation pressure
P_o	Atmospheric pressure
Z	Scaled coordinate along cone axis, equal to $x/U_D t$
λ_o	Value of z for which particle velocity vanishes in Taylor wave
σ	Mass per unit area of liner material
x, y	Cartesian coordinates of typical liner element
h	height of cone
ϕ	Bending angle (twice Taylor angle)
ϕ_N	Nth iteration of bending angle
b	Constant used in bending angle formula
τ	Time taken for liner element to hit axis
β_c	Critical angle at which the shock wave at the stagnation point becomes detached
c	Velocity of sound in liner material

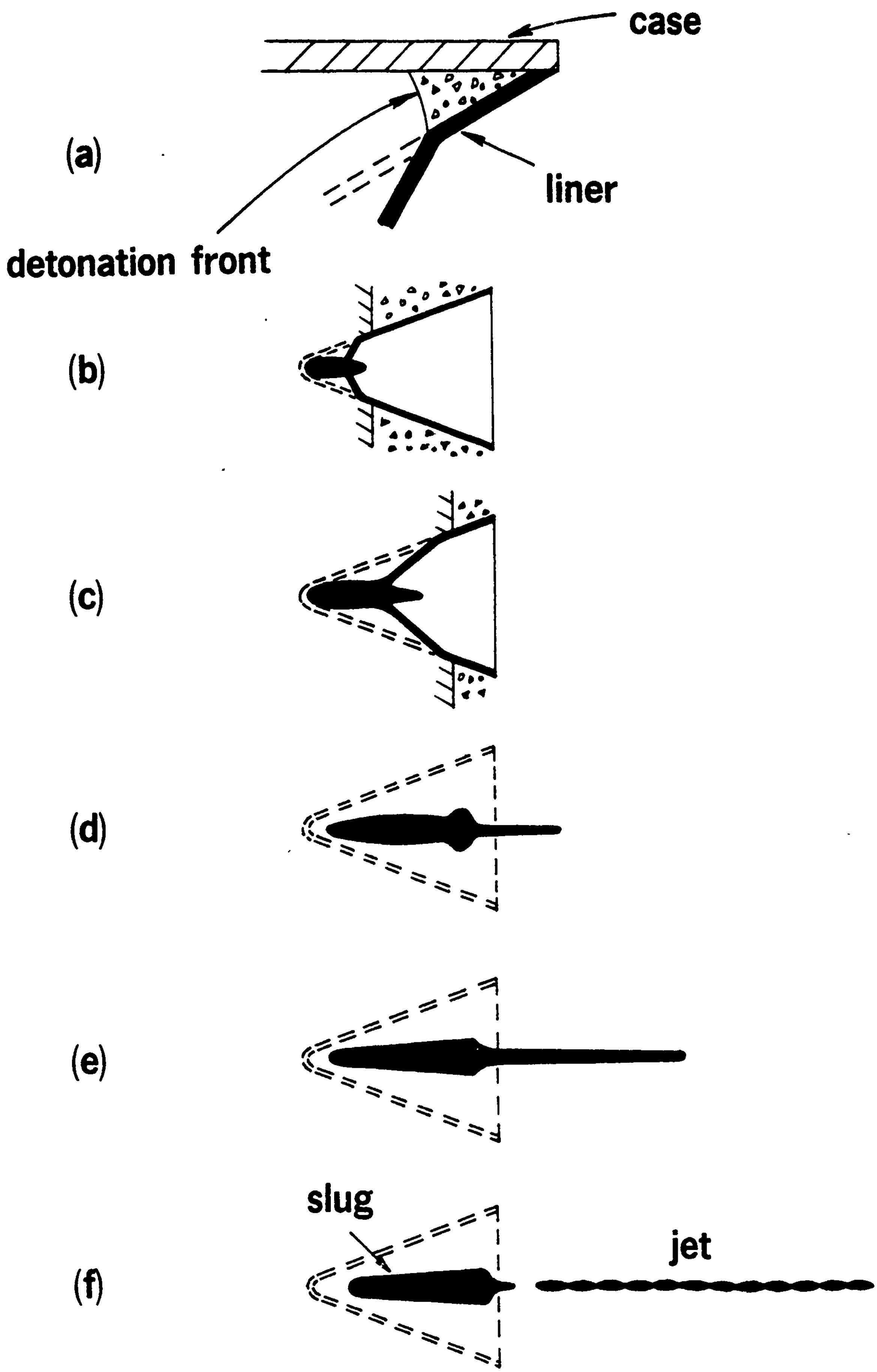

Figure 1. Schematic illustration of the progressive stages of liner collapse to form jet and slug.

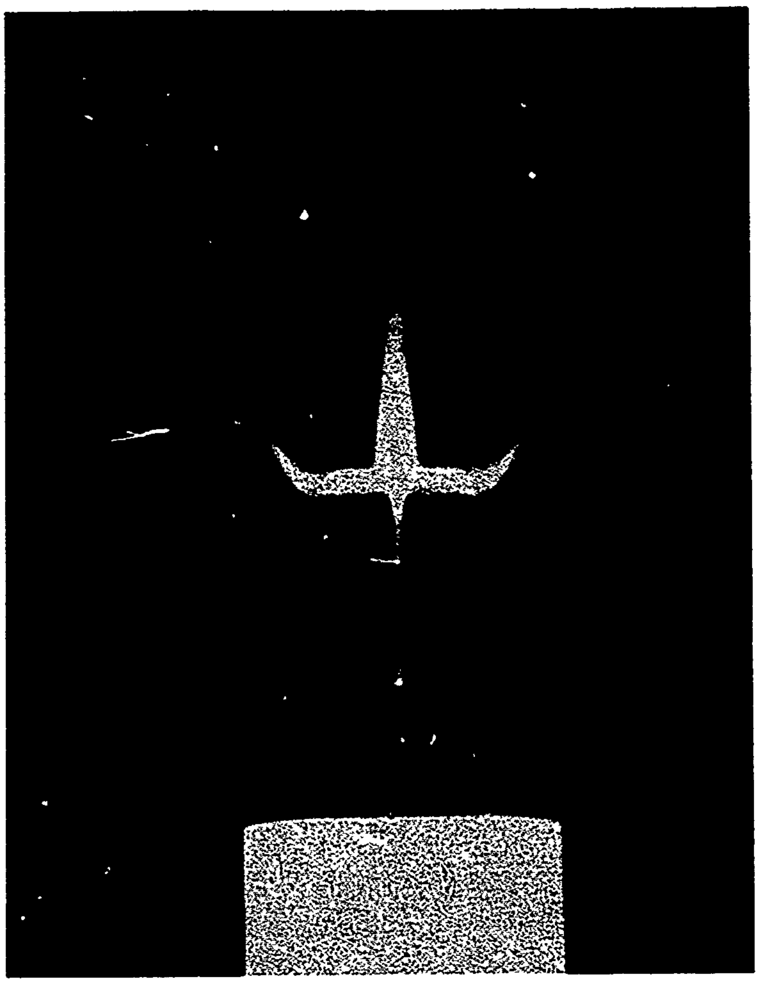

Figure 2. Flash radiograph of the collapse of the MRL 38 mm standard shaped charge 18.5 µs after initiation of detonation.

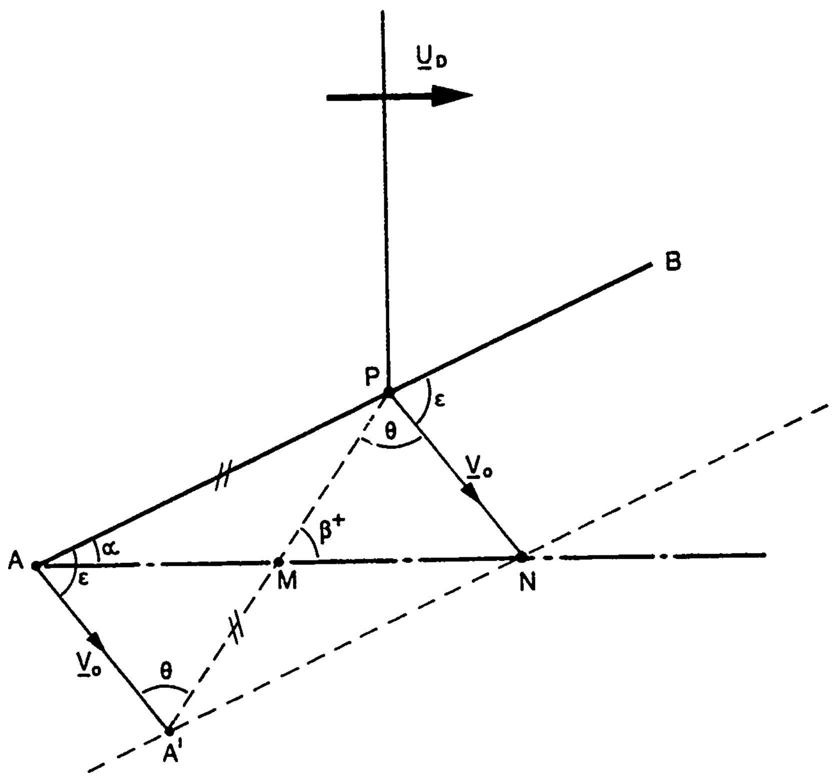

Figure 3. Diagram for determining direction of collapse velocity $\underline{V}_o$. (Adapted from reference 13.)

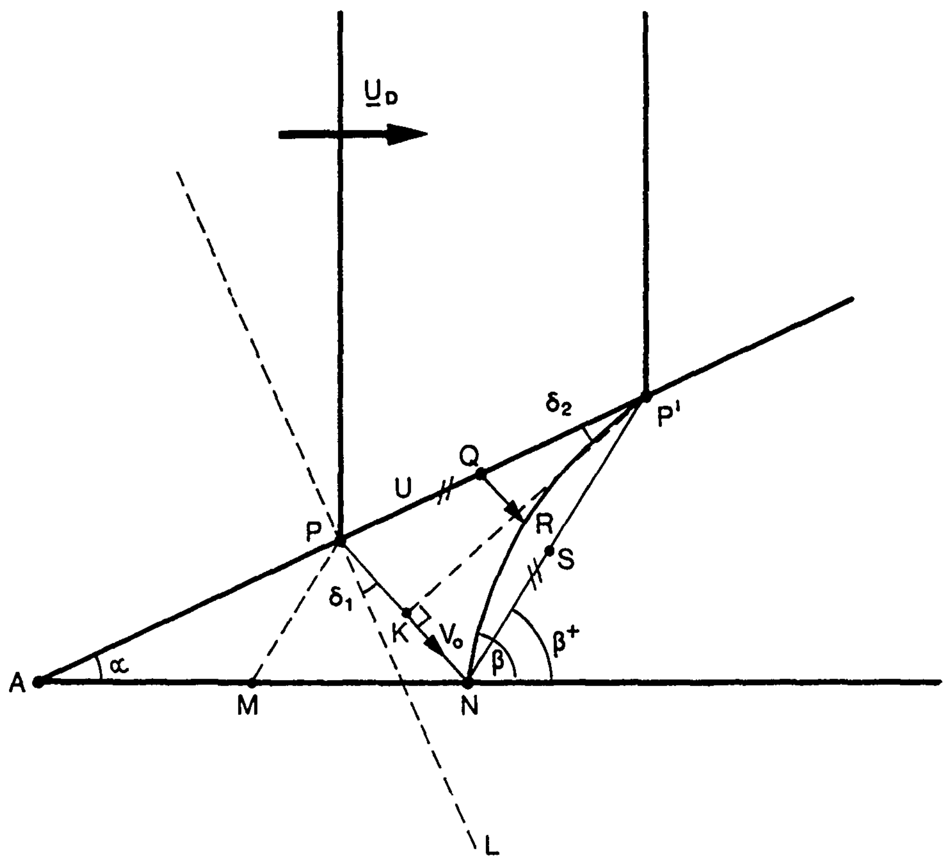

Figure 4. Diagram for determining Taylor angle formula (from reference 12).

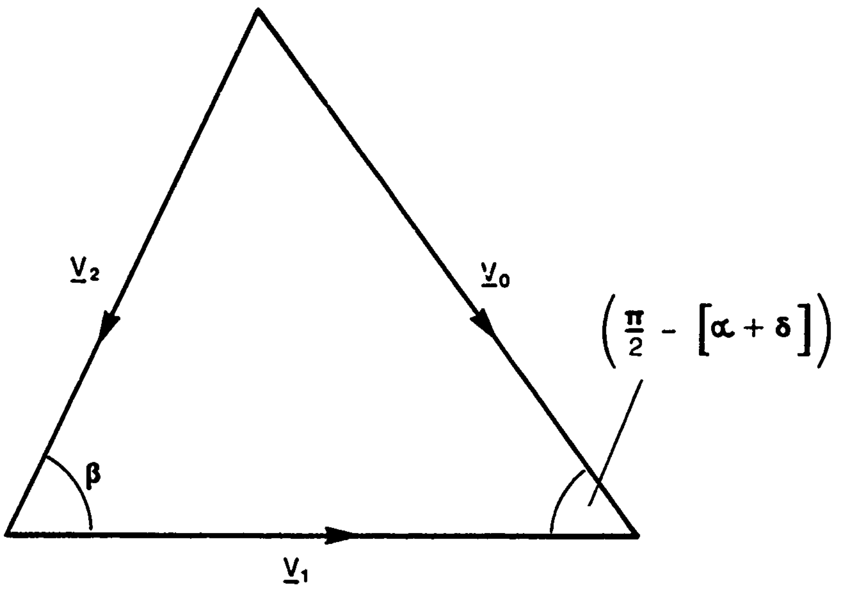

Figure 5. Relationship between velocities $\underline{V}_o$, $\underline{V}_1$ and $\underline{V}_2$ at the moving junction. (From reference 12.)

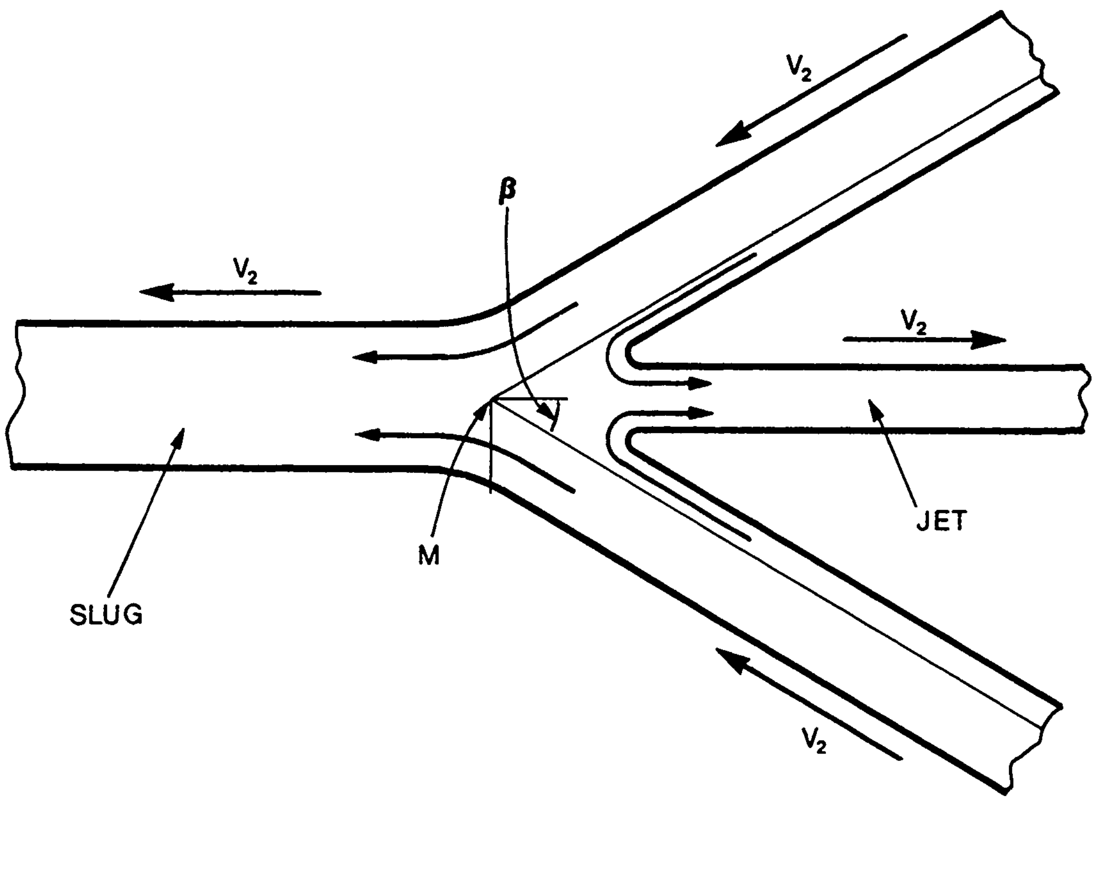

Figure 6. Formation of jet and slug by collision of two fluid jets. Adapted from referencer 13.

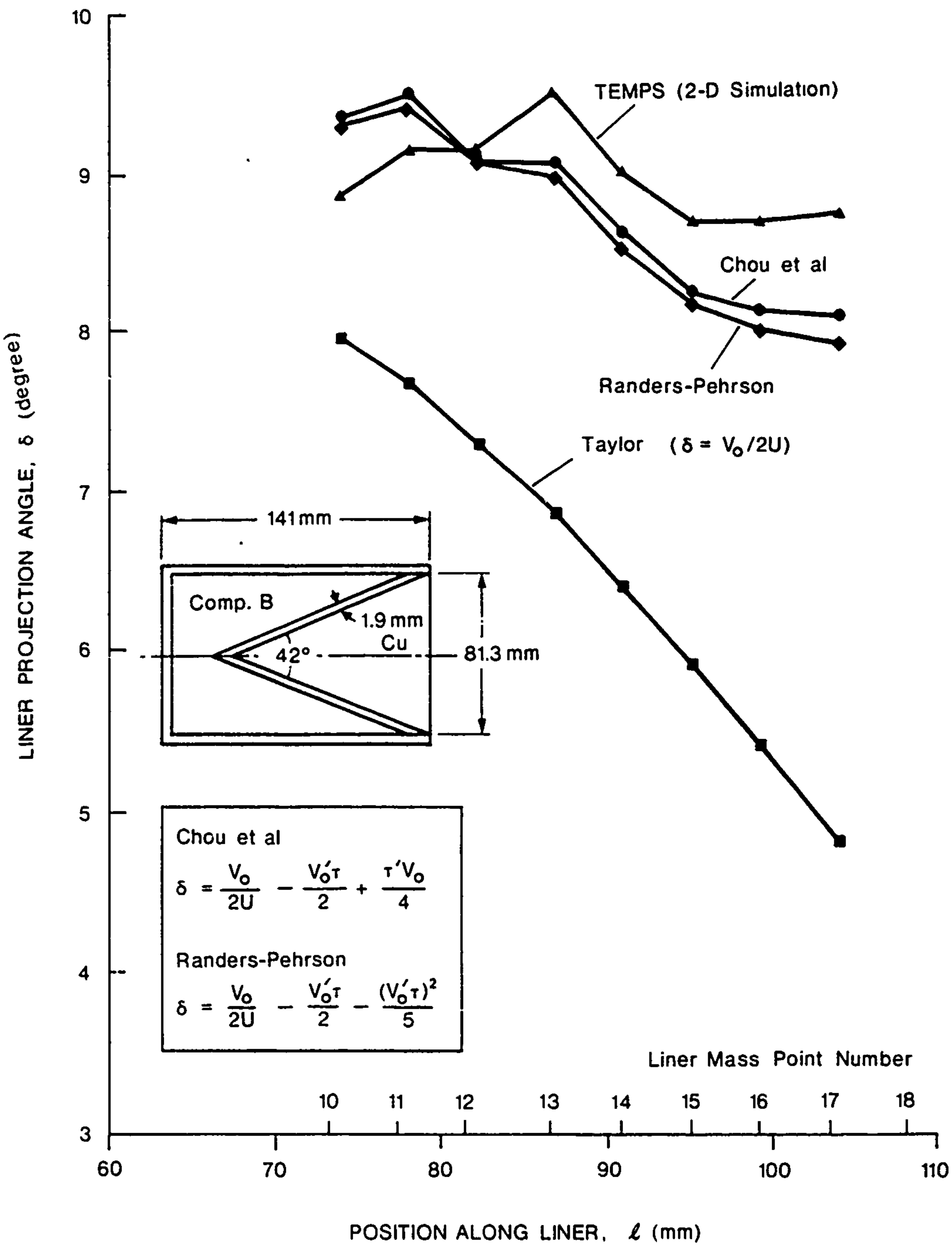

Figure 7. Comparison of the projection angle δ from formulas and two-dimensional calculations. Taken from reference 16.

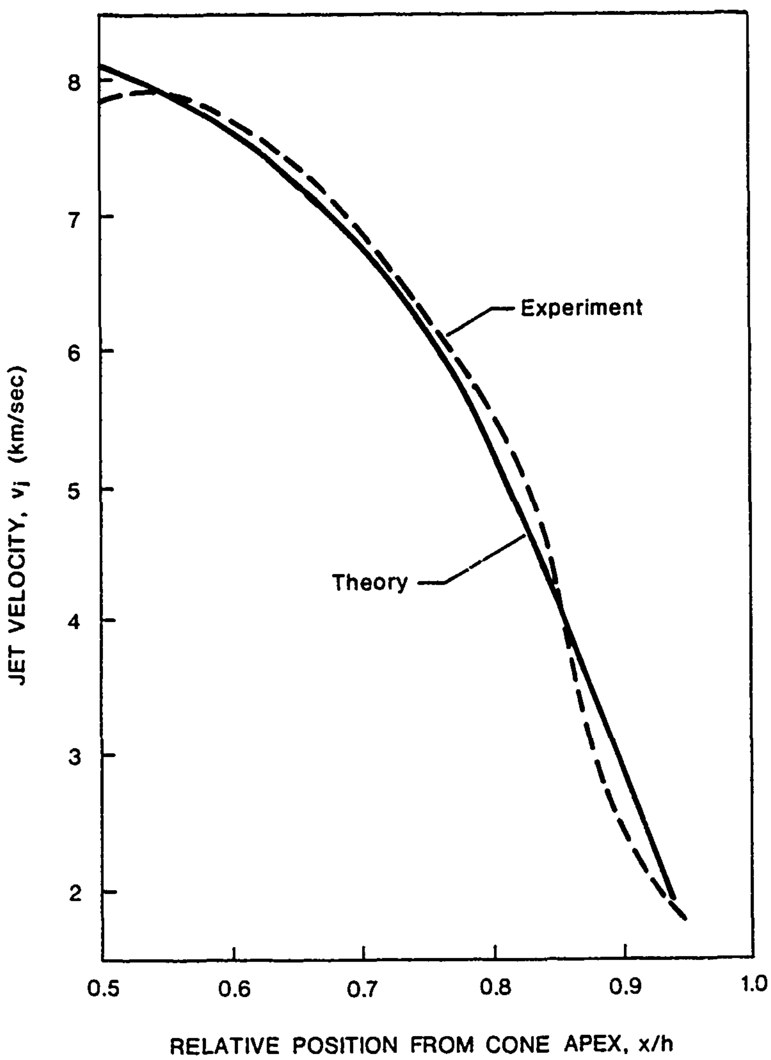

Figure 8. Experimental values of velocity gradient in jet (from reference 26) compared with calculations based on the Richter approximation for the projection angle δ (from reference 25).